# Dead Heat: The 2006 Public Choice Society Election 

Steven J. Brams<br>Department of Politics<br>New York University<br>New York, NY 10003<br>USA<br>steven.brams@nyu.edu<br>Michael W. Hansen<br>Department of Mathematics<br>Harvey Mudd College<br>Claremont, CA 91711<br>mwhansen@hmc.edu

Michael E. Orrison
Department of Mathematics
Harvey Mudd College
Claremont, CA 91711
orrison@hmc.edu


#### Abstract

In 2006, the Public Choice Society chose a new president using approval voting. There were five candidates, and the election was extremely close. We indicate the sources of support of the different candidates, based in part on spectral analysis, by voters who cast between one and five votes. Using preference information that was also gathered, we show that two candidates different from the approval voting winner, including the apparent Condorcet winner, might have won under different voting systems. Because most voters did not indicate their complete preference rankings, however, these differences are hardly robust, especially since the outcome was essentially a dead heat.


Keywords: Public Choice Society; approval voting; spectral analysis; Condorcet winner;
Borda count; Hare system.

## Dead Heat: The 2006 Public Choice Society Election

## Introduction

For the first time in its nearly 40-year history, the Public Choice Society (PCS) asked its members to vote on a new president for the term 2006-2008. Previous presidents had been selected every two years by an executive committee, mostly comprising former presidents of the PCS.

When one of us (Brams), who was president in 2004-2006, proposed at the 2005 annual meeting that members elect the next president, the executive committee concurred. The committee also supported the use of approval voting (AV) as the method of election if there were more than two candidates. The PCS joins about a dozen other professional societies, some with membership rosters numbering in the tens of thousands, that have adopted AV in the past 20 years (Brams and Fishburn, 2005).

The executive committee decided to continue the practice of alternating between economists and political scientists as president. Because 2006-2008 was the turn of economists, only economists would be eligible as candidates in the presidential election. Nominations were solicited from the membership in the fall of 2005, and five candidates agreed to run (four nominees declined).

Prior to the 2006 election, the PCS was informally defined by the people who came to its annual meeting and subscribed to, or wrote articles for, Public Choice, the unofficial journal of the PCS. To give official status to membership in PCS, all those on the mailing list (now an e-mail list) were asked to pay a nominal fee of $\$ 30$ to become members ( $\$ 15$ for students), which would make them eligible to vote in the presidential election.

Relatively few became members and sent in their ballots prior to the December 2005 election deadline. (Many more became members at, or just prior to, the 2006 annual meeting in New Orleans, March 31 April 2, in part because their registration fees were reduced by the amount of the membership fee.) In the end, 37 people became members and voted before the deadline.

Under AV, voters can vote for as many candidates as they like; the candidate with the most approval votes wins. On the AV ballot, voters were also asked, "in the spirit of research on public choice," to participate in an experiment by ranking the five candidates from best to worst.

The ballot instructions made clear, however, that the rankings would have no effect on the election
outcome. But they enabled us to incorporate preference information into our analysis that the approval votes, by themselves, did not reveal. We begin by reporting the AV results and the patterns of support that different sets of candidates received.

## The AV Outcome

Because one voter, in ranking the five candidates, did not indicate approval of any (perhaps an oversight), we based the AV outcome on the 36 ballots that indicated approval of between one and five candidates, whom we identify as $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$, and E . As is evident from the table below, the approval totals could hardly have been closer, with a tie for winner and the third candidate just one vote behind (later we will indicate how a winner was chosen):

| Candidates | A | B | C | D | E |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Votes | $17(23 \%)$ | $16(22 \%)$ | $17(23 \%)$ | $14(19 \%)$ | $9(12 \%)$ |

The 73 approval votes cast by the 36 voters translate into the voters' approving of an average of 2.03 candidates. The numbers of voters casting different numbers of votes is as follows:

| Approvals | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| \# of voters | $15(42 \%)$ | $10(28 \%)$ | $7(19 \%)$ | $3(8 \%)$ | $1(3 \%)$ |

Well over half the voters ( 21 , or 58 percent) voted for more than one candidate. What motivated one voter to approve of all five candidates, which raised everybody's total by one vote but had no effect on the AV outcome, is unclear. (Perhaps he or she lacked information about the candidates, was genuinely indifferent among them, or just wanted to show support for the PCS.)

It is instructive to examine the sources of approval of the five candidates. We show below the number of approval votes that each of the five candidates received from voters casting between one and five votes:

| Candidate | A | B | C | D | E |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1-voters | 2 | 4 | 4 | 3 | 2 |
| 2-voters | 5 | 4 | 3 | 5 | 3 |
| 3-voters | 6 | 4 | 6 | 3 | 2 |
| 4-voters | 3 | 3 | 3 | 2 | 1 |
| 5-voter | 1 | 1 | 1 | 1 | 1 |
| Total | 17 | 16 | 17 | 14 | 9 |

Observe that candidates D and E received more than half their support from 1-voters and 2-voters, whereas candidates A and C received more than half from 3-voters, 4 -voters, and 5 -voters. In between, candidate B drew equally from both sets of voters. Thus, the voters who were more expansive in their approval helped the biggest vote-getters ( A and C ).

The question of who shares support with whom can be gleaned from $n$-tuples of candidates who were approved of by the 21 voters who voted for more than one candidate. They are shown below in decreasing order of approval, with the number of voters approving of each subset of 2,3 , and 4 candidates shown in parentheses:

2-voters: DE (3); $\mathrm{AB}(2) ; \mathrm{AC}(2) ; \mathrm{AD}(1) ; \mathrm{BC}(1) ; \mathrm{BD}(1)$
3-voters: ABC (3), ACD (2); ACE (1); BDE (1)

## 4-voters: $\mathrm{ABCD}(2) ; \mathrm{ABCE}$ (1)

The four pairs of candidates to share the most support - by being members of subsets approved of by either 2 -voters, 3 -voters, or 4 -voters - are AC (11 voters), AB (8 voters), BC (7 voters), and AD (5 voters).

Candidate A had more shared support than any other candidate. While candidate E is not a member of any of the top pairs, it is noteworthy that among 2 -voters, DE has the most approval. We speculate that D and E were perceived to be theorists and garnered most of their support from like-minded voters. Spectral analysis later will further illuminate the shared support of candidates.

## What Do the Rankings Tell Us?

Of the 37 voters, 13 ranked all five candidates, whereas 10 indicated only a first choice or a tied first choice. For these 10 voters and the 14 that gave different partial rankings, we assumed candidate I is preferred to candidate $\mathbf{J}$ if and only if (i) I is ranked higher than J or (ii) I is approved and J is not approved (these two criteria were always consistent); otherwise, we assumed indifference between the candidates. Using these rankings, we calculated what the outcomes would have been under four different voting systems: ${ }^{1}$

1. Plurality voting would have chosen candidate $D$. We assumed that voters would vote for their first choices; if they ranked more than one candidate first, this vote would be split evenly among all their first choices. Surprisingly, candidate D, the fourth-place AV candidate, would have won decisively with 11.20 votes, followed by candidates (A, C, B, E) with ( $8.53,7.53,7.20,2.53$ ) votes.
2. A Condorcet voting system-one that always elects a Condorcet winner if one exists-would have chosen candidate $C .{ }^{2}$ A Condorcet winner is a candidate who is socially preferred to all other candidates in pairwise comparisons. Candidate C, one of the two tied AV winners, would have defeated candidates ( $\mathrm{D}, \mathrm{B}, \mathrm{A}, \mathrm{E}$ ) by margins of $(1,3,6,11)$ votes.
3. The Borda count would have chosen candidate C. Under the Borda count, candidates obtain (4, 3, 2, $1,0)$ points for being ranked $\left(1^{\text {st }}, 2^{\text {nd }}, 3^{\text {rd }}, 4^{\text {th }}, 5^{\text {th }}\right)$. We assumed that ties for ranks give candidates the average of points for these ranks. Following candidate C ( 84.5 points) are candidates A ( 81.5 points), $\mathrm{D}(79.5$ points), B ( 73.5 points), and E ( 51.0 points).
4. The Hare system of single transferable vote (STV) would have chosen candidate D. STV successively eliminates candidates with the fewest first-choice votes and transfers them to second and, if necessary, lower choices. Candidate E would have been eliminated first, followed by candidate B , and then candidates C and A together (because they tied for lowest).

In sum, none of the four alternative voting systems would have chosen one of the two tied AV winners (candidate A). Two would have chosen candidate C , the other tied AV winner, and two would have chosen candidate D , the fourth-place AV winner.

We hasten to add that the voters might well have behaved differently if one of the alternative voting systems had been used. Thus, these reconstructions must be taken with a grain of salt. Nevertheless, they
indicate the sensitivity of election outcomes to the voting rules, especially in an election as close as that of the PCS.

## Spectral Analysis

One of us (Orrison) recently applied spectral analysis to U.S. Supreme Court voting data (Lawson, Orrison, and Uminsky, 2006), which is an exploratory data methodology that offers further insight into the sources of support of PCS candidates. While we will not provide mathematical details on spectral analysis, suffice it to say that it is based on linear algebra and related algebraic methods and can be used to uncover patterns in voting data (e.g., approval votes or rankings). Additionally, there are efficient algorithms that facilitate the application of spectral analysis to empirical data.

Some of the conclusions we drew from the spectral analysis are the following:

1. Candidates $A$ and $C$ on the one hand, and candidates $D$ and $E$ on the other, derive their approval from quite different supporters. This is consistent with our earlier finding that AC is the most approved pair among 2 -voters, 3 -voters, and 4 -voters, whereas DE is most approved by just the 2 -voters. Also indicative of the distinctiveness of AC and DE supporters is that none of the three 4 -voters voted for these four candidates, and only three of the seven 3-voters voted for three of the four candidates. A possible explanation is that D and E are the most theoretically oriented of the candidates, as previously noted, while A and C have a more empirical orientation.
2. Aside from candidate A, who had the most shared approval, each of the other candidates was essentially indistinguishable in terms of the support he or she received from 1-voters. The support that gave some candidates the edge, and made the difference in the AV outcome, came from voters who cast multiple approval votes.
3. Candidate $D$ stood out as the person who did relatively poorly under AV, placing fourth, but jumped to the top in first choices. We interpret this to mean that D had a significant number of strong supporters (almost one-third of the voters ranked D first), but the majority of voters probably did not know D or D's work well and so gave D middling to low ranks and little approval. At the other end of the spectrum, the candidate who came in last under AV (E) was also ranked last by more voters than any other candidate, but this may be attributable to E's being relatively unknown among PCS members.

## Who Won?

Candidate A won not because the tie in approval votes between A and C was broken by a coin toss but because the ballots we analyzed were incomplete: When the AV ballots were originally compiled by the Center for the Study of Public Choice at George Mason University, the vote totals for (A, B, C, D, E) were $(19,17,18,15,10)$. These totals gave A two more votes than reported earlier and the other candidates one more vote. It seems that at least two ballots were lost or misplaced when they were sent to us, so our analysis reflects incomplete returns and, consequently, does not give a complete picture of how the candidates fared and their sources of support.

But even though the election was not, in fact, a dead heat, it is apparent that different voting procedures may have given different results. We do not claim that AV is the best way to aggregate preferences, because there are valid arguments that can be made for other voting procedures, including those that combine approval and preference (Brams and Sanver, 2006) or seem better suited for electing committees or councils (Potthoff and Brams, 1998; Brams, Kilgour, and Sanver, 2006).

## Conclusions

We conclude on a practical note. Even for voters as sophisticated as PCS members, only a minority ranked all five candidates, which we suspect would also have been true if one of the ranking methods had been used instead of AV. It seems that most voters had difficulty drawing distinctions among the five candidates, probably because they lacked information about all of them.

AV does not ask voters to make fine distinctions. Instead, it asks them to draw a line, wherever they choose, between approved and disapproved candidates. In this case, AV found a consensus choice (candidate A) - one acceptable to the most voters - that the ranking systems failed to do (at least for our incomplete data set).

## References

Brams, Steven J., and Peter C. Fishburn (2005). "Going from Theory to Practice: The
Mixed Success of Approval Voting." Social Choice and Welfare 25, no. 2: 457-
474.

Brams, Steven J., and Peter C. Fishburn (2002). "Voting Procedures." In K. J. Arrow, A.
K. Sen, and K. Suzumura (eds.), Handbook of Social Choice and Welfare, vol. 1.

Amsterdam: Elsevier Science, pp. 173-206.
Brams, Steven J., D. Marc Kilgour, and M. Remzi Sanver (2006). "A Minimax Procedure for Electing Committees." Preprint, New York University.

Brams, Steven J., and M. Remzi Sanver (2006). "Voting Procedures That Combine Approval and Preference." Preprint, New York University.

Dummett, Michael (1984). Voting Procedures. Oxford, UK: Clarendon Press.
Lawson, Brian L., Michael Orrison, and David T. Uminsky (2006). "Spectral Analysis of the Supreme Court." Mathematics Magazine (forthcoming).

Lijphart, Arend, and Bernard Grofman (eds.) (1984). Choosing an Electoral System:
Issues and Alternatives. New York: Praeger.
Potthoff, Richard F., and Steven J. Brams (1998). "Proportional Representation:
Broadening the Options." Journal of Theoretical Politics 10, no. 2 (April):
147-178.
Regenwetter, Michel, Bernard Grofman, A. A. J. Marley, and Ilia Tsetlin (2006).
Behavioral Social Choice: Probabilistic Models, Statistical Inference, and.

Applications. Cambridge, UK: Cambridge University Press.
Saari, Donald G. (2001). Decisions and Elections: Explaining the Unexpected. New York: Cambridge University Press.

